

Quantum Information with Solid-State Devices

VO I41.A55

SS2016

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Lecture 2



I Basic Concepts

qubit/quantum bit
Bloch sphere
Rabi oscillation
open quantum systems
density matrix
decoherence/dephasing
Lindblad equation
Ramsey oscillation
echo techniques

Pauli Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Pauli matrices σ_i are hermitian and unitary. Eigenvalues are -1,1.

$$\sigma_i \sigma_j = \delta_{ij} + i \epsilon_{ijk} \sigma_k$$

$$\begin{aligned}\sigma_x \sigma_y &= -\sigma_y \sigma_x = i \sigma_z \\ \sigma_x^2 &= \sigma_y^2 = \sigma_z^2 = \mathbf{1} \\ [\sigma_x, \sigma_y] &= 2i \sigma_z \\ \{\sigma_x, \sigma_y\} &= 0\end{aligned}$$

$$\begin{aligned}\text{Tr}(\sigma_i) &= 0 \\ \det(\sigma_i) &= -1\end{aligned}$$

$$\sigma_+ = \frac{1}{2} (\sigma_x + i \sigma_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

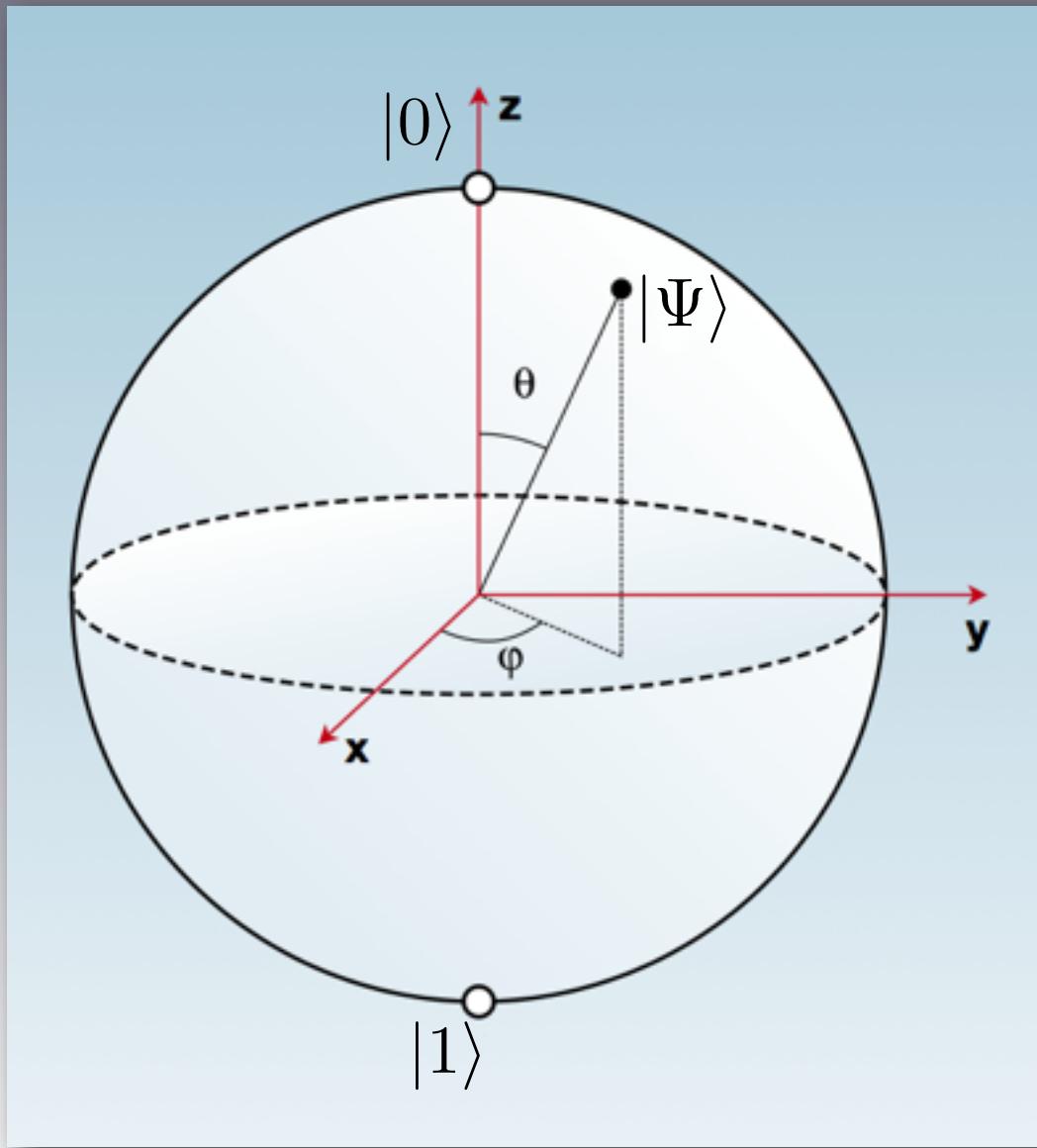
$$\sigma_- = \frac{1}{2} (\sigma_x - i \sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_x = \sigma_+ + \sigma_- \quad \sigma_y = \frac{1}{i} (\sigma_+ - \sigma_-)$$

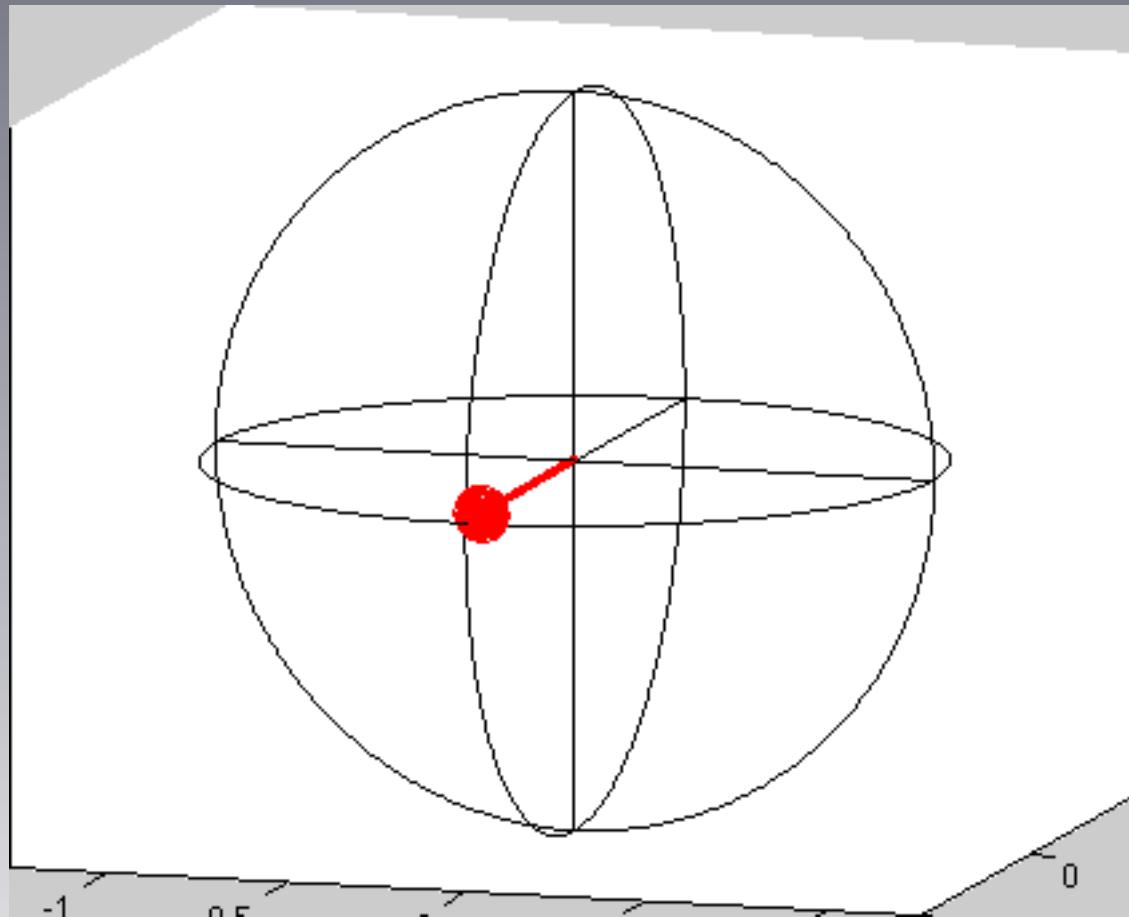
$$\sigma_+ \sigma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \sigma_- \sigma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{1} = \sigma_+ \sigma_- + \sigma_- \sigma_+ \quad \sigma_z = \sigma_+ \sigma_- - \sigma_- \sigma_+ \quad 2\sigma_+ \sigma_- = \mathbf{1} + \sigma_z$$

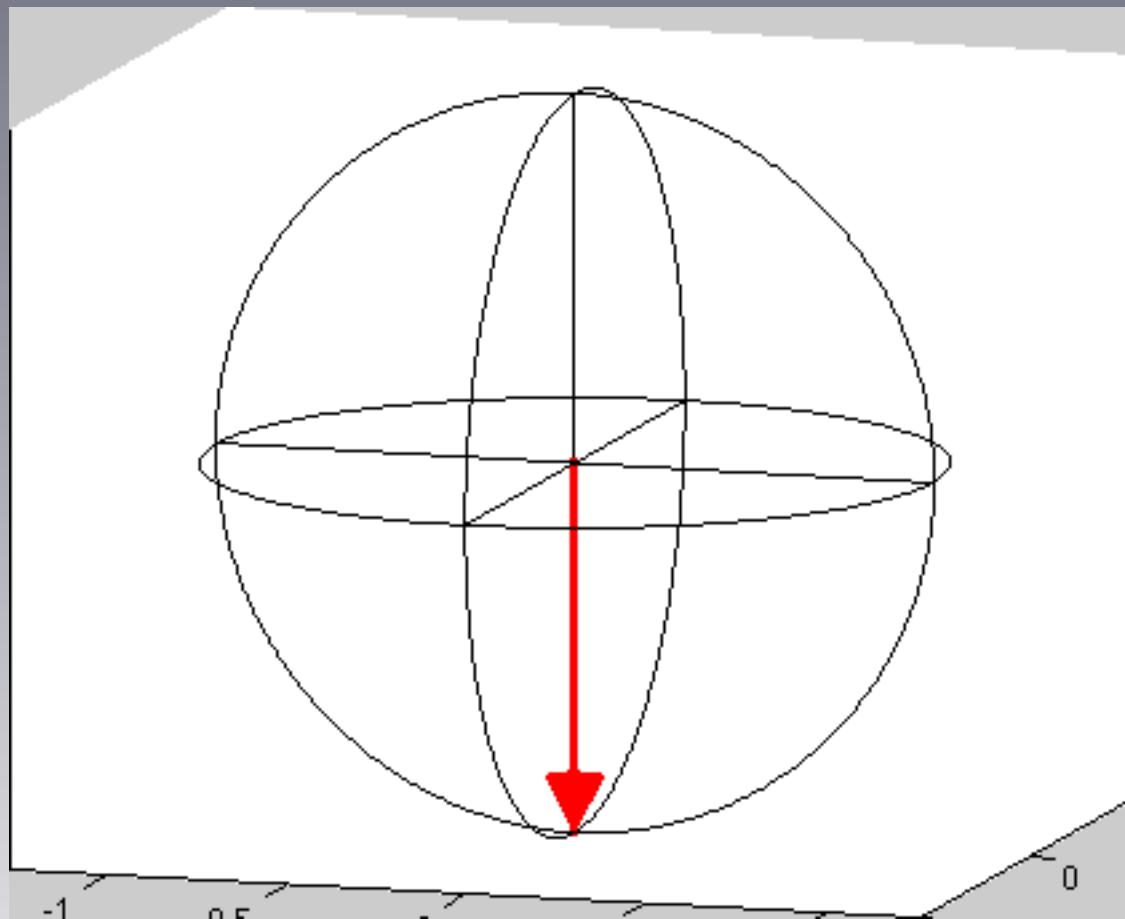
Bloch Sphere



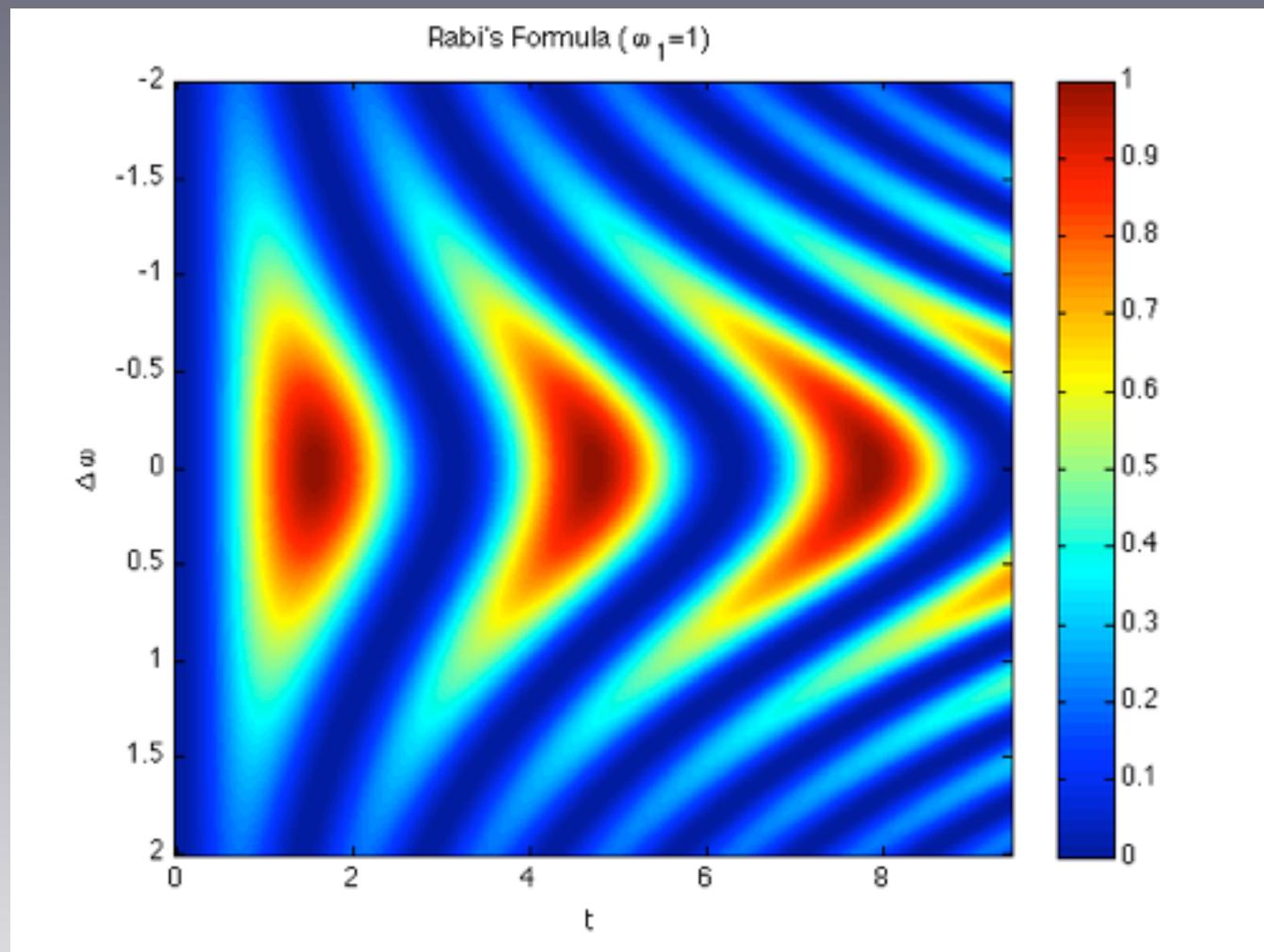
Larmor Precession



Rabi

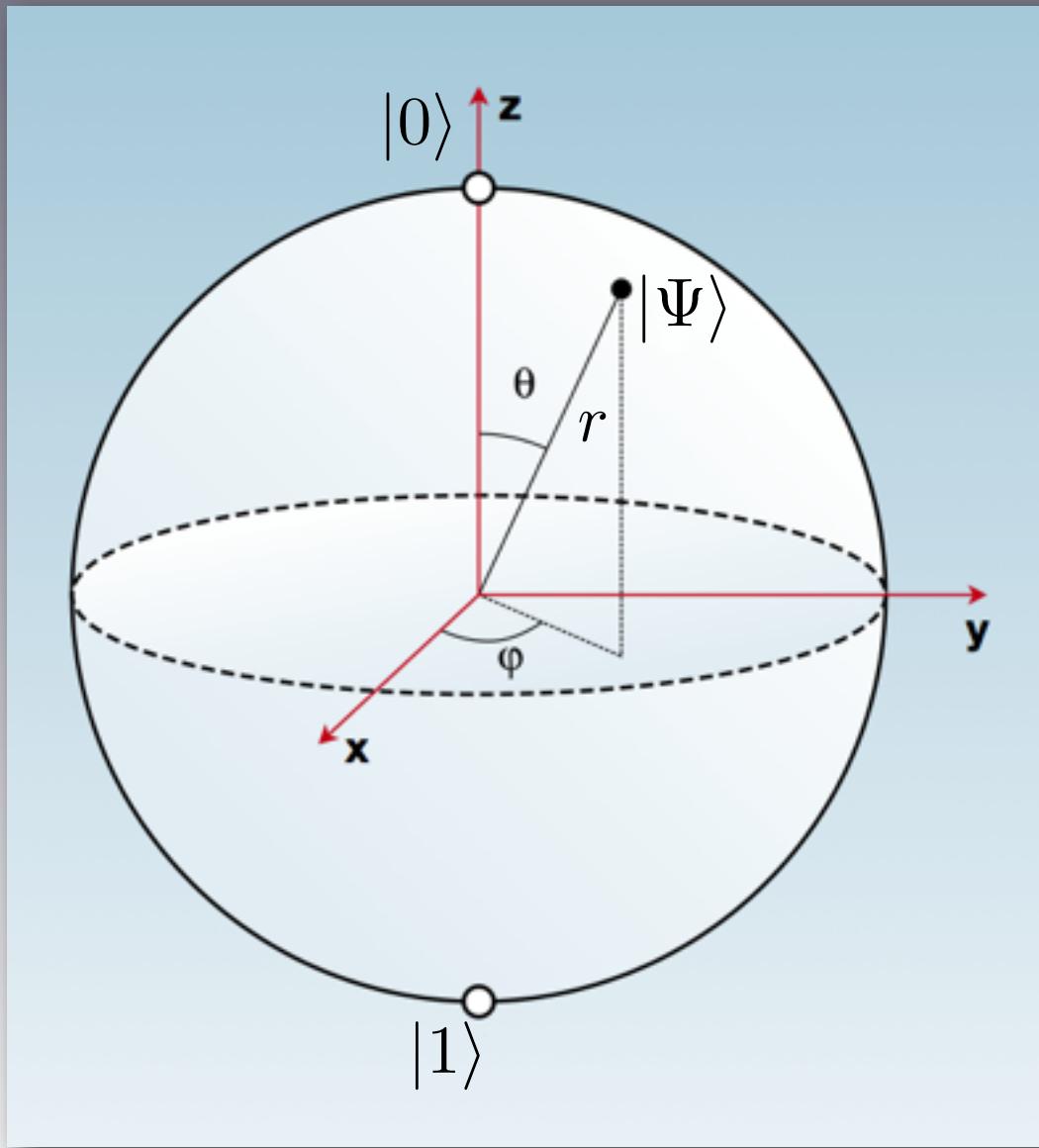


Rabi's Formula



$$P_1(t) = \frac{\omega_1^2}{\omega_1^2 + \Delta\omega^2} \sin(\sqrt{\omega_1^2 + \Delta\omega^2}t)^2$$

Bloch Sphere



$$x = r \sin(\theta) \cos(\varphi)$$

$$y = r \sin(\theta) \sin(\varphi)$$

$$z = r \cos(\theta)$$

References Lecture 2

Modern Quantum Mechanics
J. J. Sakurai
Addison Wesley

Quantum Computation and Quantum Information
Michael A. Nielsen, Isaac L. Chuang
Cambridge University Press